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OPTIMAL TAXATION OF ENTREPRENEURIAL CAPITAL WITH PRIVATE INFORMATION

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ABSTRACT

This paper studies optimal taxation of entrepreneurial capital and financial assets in economies with private information. Returns to entrepreneurial capital are risky and depend on entrepreneurs' hidden effort. It is shown that the idiosyncratic risk in capital returns implies that the intertemporal wedge on entrepreneurial capital that characterizes constrained-efficient allocations can be positive or negative. The properties of optimal marginal taxes on entrepreneurial capital depend on the sign of this wedge. If the wedge is positive, the optimal marginal capital tax is decreasing in capital returns, while the opposite is true when the wedge is negative. Optimal marginal taxes on other assets depend on their correlation with idiosyncratic capital returns. The optimal tax system equalizes after tax returns on all assets, thus reducing the variance of after tax returns on capital relative to other assets. If entrepreneurs are allowed to sell shares of their capital to outside investors, returns to externally owned capital are subject to double taxation- at the level of the entrepreneur and at the level of the outside investors. Even if entrepreneurs can purchase private insurance against their idiosyncratic risk, optimal asset taxes are essential to implement the constrained-efficient allocation if entrepreneurial portfolios are private information.

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1. Introduction

This paper studies the optimal taxation of entrepreneurial capital and financial assets in private information economies. The optimal setting of taxes on capital income is a classic question in macroeconomics and public finance. The focus on entrepreneurial capital is motivated by the fact that it accounts for a disproportionately large fraction of household wealth and economy-wide capital in the US economy. Based on the Survey of Consumer Finances (SCF), Gentry and Hubbard (2000) estimate that approximately 38% of assets of the household sector are held by entrepreneurs. Using the PSID, Quadrini (1999) documents that entrepreneurial assets account for 46% of household wealth. Moscowitz and Vissing-Jorgensen (2002) identify entrepreneurial capital with private equity, and they document that its value is similar in magnitude to public equity from SCF data.

The premise of the analysis is that the main source of risk for entrepreneurs is *capital risk* and that incentive problems due to *informational frictions* play a central role in entrepreneurial activity. The approach used to derive the optimal tax system builds on the seminal work of Mirrlees (1971), and extends it to a dynamic setting. In the first step, we characterize the constrained-efficient allocation, which solves a planning problem subject to the incentive compatibility constraints resulting from the informational frictions. We then construct a set of taxes that implements such an allocation as a competitive equilibrium. The only a priori restriction is that taxes must depend on observables. The resulting tax system optimizes the trade-off between insurance and incentives[1].

The recent literature on dynamic optimal taxation with private information, as Albanesi and Sleet (2006) and Kocherlakota (2005b), has focussed on economies with idiosyncratic risk in labor income, thus abstracting from the incentive problems that are potentially prevalent for entrepreneurial capital. Macroeconomic studies of optimal capital income taxes have also devoted little attention to the taxation of financial assets other than capital and to whether capital should be taxed at the firm or at the investor level. Yet, the empirical public finance literature has documented substantial differences in the tax treatment of different forms of capital income and strong response of household portfolio composition and firms's investment decisions to this differential tax treatment[2]. The contribution of this paper is to study optimal allocations with idiosyncratic capital income risk and private information, derive the implications for optimal taxes on entrepreneurial capital and other assets, as well as explore the question of optimal tax incidence and investigate the complementarity between optimal tax systems and private contracts.

The analysis is based on a model of entrepreneurial activity where returns to capital

[1] This recent literature is summarized in Kocherlakota's (2005a) excellent review.

[2] See Gordon and Slemrod (1988), Gordon (2003), Poterba (2002) and Auerbach (2002)

are risky and positively depend on entrepreneurs' effort, which is *private information.* Entrepreneurial investment and capital returns are assumed to be observable. The dependence of capital returns on effort implies that capital is agent specific and generates idiosyncratic capital risk. The unobservability of effort gives rise to a dynamic moral hazard problem.

The constrained-efficient allocation solves the problem of a planner who allocates investment and consumption across time and states to maximize the agents' ex ante lifetime utility, subject to a resource constraint and incentive compatibility constraints. This allocation displays a wedge between the entrepreneurs' intertemporal marginal rate of substitution and the economy-wide marginal rate of transformation. Golosov, Kocherlakota and Tsyvinski (2003) show that this wedge is positive for a large class of private information economies with idiosyncratic labor risk. However, this *aggregate* intertemporal wedge is not related to the entrepreneurs' incentives to exert effort with idiosyncratic capital risk, since the individual intertemporal rate of transformation differs from the aggregate. Hence, we introduce the notion of an *individual* intertemporal wedge, which properly accounts for the agent specific nature of entrepreneurial capital returns. We show that the negative covariance between marginal utility of consumption and entrepreneurial capital returns implies that the individual intertemporal wedge can be *negative.* The intuition for this result is simple. More capital increases an entrepreneur's consumption in the bad states and provides insurance, which has an adverse effect on incentives. On the other hand, expected capital returns are increasing in entrepreneurial effort, hence the benefits from increasing effort are increasing in the level of capital. We show that the second effect dominates when the spread in capital returns is sufficiently large or when the variability of consumption across states is small at the constrained-efficient allocation. In this case, more capital relaxes the incentive compatibility constraint and increases entrepreneurial effort.

To study optimal taxes, we consider how to implement the constrained-efficient allocation in a setting where agents can trade in competitive markets and are subject to taxes that influence their budget constraints. We examine three different market structures. A market structure specifies the distribution of ownership rights, the information structure and the feasible trades between agents. These arrangements are treated as exogenous. A tax system implements the constrained-efficient allocation if such an allocation arises as the competitive equilibrium under this tax system for the assumed market structure. In all of the market structures we consider, the level of investment chosen by the entrepreneurs and capital returns are assumed to be observable.

In the first market structure, entrepreneurs trade bonds. We show that the optimal marginal tax on entrepreneurial capital is increasing in income, when the intertemporal wedge is negative, while it is decreasing in income when the intertemporal wedge is positive. The intertemporal wedge on the risk-free bond is always positive and higher than the intertemporal wedge on capital, and the marginal tax on bonds is decreasing in income. The optimal tax system equates the after tax return on all assets in each state. This implies that entrepreneurial capital is subsidized in the low income states, relative to other assets, irrespective of the sign of the intertemporal wedge. The sign of the intertemporal wedge and optimal marginal taxes for risky securities depend on the correlation of their returns with idiosyncratic risk. These results provide a clear prescription for *differential asset taxation*. By contrast, as shown in Albanesi and Sleet (2006) and Kocherlakota (2005b), the optimal marginal tax on capital income is decreasing in income in economies with labor risk, and this property holds independently of the nature of the asset.[3]

The second market structure allows entrepreneurs to sell shares in their own capital and buy shares of other entrepreneurs' capital. Each entrepreneur can be viewed as a firm, so that this arrangement introduces an equity market. The optimal tax system for this market structure embeds a prescription for optimal *double taxation of capital*- at the firm level, through the marginal tax on entrepreneurial earnings, and at the investor level, through a marginal tax on stocks returns. Specifically, it is necessary that the tax on earnings be "passed on" to stock investors via a corresponding reduction in dividend distributions to avoid equilibria in which entrepreneurs sell all their capital to outside investors. In such equilibria, an entrepreneur exerts no effort and thus it is impossible to implement the constrained-efficient allocation. Since, in addition, marginal taxation of dividends received by outside investors is necessary to preserve incentives for the usual reasons, earnings from entrepreneurial capital are subject to double taxation.

Entrepreneurial asset holdings have been assumed to be observable so far but, they need not be known to the government to administer the optimal tax system, since the optimal taxes do not depend on the level of asset holdings. This motivates the third market structure, in which insurance companies offer incentive compatible contracts to the entrepreneurs, but they cannot observe their asset holdings. We assume that the government also cannot observe asset holdings and that assets are traded via financial intermediaries who collect taxes at the source, according to the schedule prescribed by the government. We show that, absent any taxes on asset holdings, the private insurance contracts do not implement the constrained-efficient allocation. On the other hand, by appropriately setting marginal asset taxes, the government can relax the more severe incentive compatibility constraint that arises in the contracting problem between private insurance companies and entrepreneurs, due entrepreneurs' unobserved holdings of financial assets. Hence, only under the optimal tax system private insurance contracts implement the constrained-efficient allocation with observable consumption. This finding has important implications for the role of the government in implementing allocations. Even under the same informational constraints as private insurance companies, the government can influence the portfolio choices of entrepreneurs through the

[3] These studies stop short of allowing agents to trade more than one asset.

tax system. This result is related to Golosov and Tsyvinski (2004), who analyze fiscal implementations in a Mirrleesian economy with unobserved consumption. They show that private insurance contracts do not implement constrained-efficient allocations in that setting, because private insurers fail to internalize the effect of the contracts they offer on the equilibrium price of unobservable bond trades. A linear tax on capital can instead ameliorate this externality. Here, instead, under the optimal tax system, the government implements the constrained-efficient allocation with observable consumption, despite the fact that in the competitive equilibrium consumption is not observed.

This paper is related to Farhi and Werning (2005) who study optimal estate taxation in an overlapping generation economy with private information. They find that the intertemporal wedge is negative if agents discount the future at a higher rate than the planner. Grochulski and Piskorski (2005) study optimal wealth taxes in economies with risky human capital, where human capital and idiosyncratic skills are private information. Cagetti and De Nardi (2004) explore the effects of tax reforms in a quantitative model of entrepreneurship with endogenous borrowing constraints. Finally, Angeletos (2006) studies competitive equilibrium allocations in a model with exogenously incomplete markets and idiosyncratic capital risk. He finds that, if the intertemporal elasticity of substitution is high enough, the steady state level of capital is lower than under complete markets.

The plan of the paper is as follows. Section 1 present the economy and studies constrained-efficient allocations and the incentive effects of capital. Section 2 investigates optimal taxes. Section 3 concludes.

2. A Model

The economy is populated by a continuum of unit measure of agents, which we will refer to as entrepreneurs. Each entrepreneur lives for two periods and her lifetime utility given by:

$$U = u(c_0) - v(e) + \beta u(c_1),$$

where, c_t denotes consumption in period $t = 0, 1$ and e denotes effort exerted at time 0, with $e \in \{0, 1\}$. We assume $\beta \in (0, 1)$, $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$, and $\lim_{c \to 0} u'(c) = \infty$.

Entrepreneurs are endowed with K_0 units of the consumption good at time 0. The distribution of initial endowments is denote with $\Psi_0(K_0)$. Entrepreneurs can operate an investment technology. If K_1 is the amount invested at time 0, the return on investment at time 1 is $R(K_1)$, where:

$$R(K_1) = K_1(1 + x),$$

and x is the random net return on capital. The stochastic process for x is:

$$x = \begin{cases} \bar{x} \text{ with probability } \pi(e), \\ \underline{x} \text{ with probability } 1 - \pi(e), \end{cases} \tag{1}$$

with $\bar{x} > \underline{x}$ and $\pi(1) > \pi(0)$. The first assumption implies that $E_1(x) > E_0(x)$, where E_e denotes the expectation operator for probability distribution $\pi(e)$. Hence, the expected returns on capital is increasing in effort.

We assume effort is *private information*, while the realized value of x, as well as its distribution, and K_0 and K_1 are *public information*.

We assume for simplicity that the distribution of initial capital is degenerate at K_0. The constrained-efficient allocation for this economy is the solution to the following problem:

$$\{e^*, K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})\} = \arg \max_{e \in \{0,1\}, K_1 \in [0, K_0],\ c_0, c_1(x) \geq 0} u(c_0) - v(e) + \beta E_e u(c_1(x)) \tag{Problem 1}$$

subject to

$$c_0 + K_1 \leq K_0,\ E_e c_1(x) \leq K_1 E_e(1+x), \tag{2}$$

$$\beta E_1 u(c_1(x)) - \beta E_0 u(c_1(x)) \geq v(1) - v(0), \tag{3}$$

where E_e denotes the expectation operator with respect to the probability distribution $\pi(e)$. The constraints in (2) stem from resource feasibility, while (3) is the incentive compatibility constraint, arising from the unobservability of effort. We will denote the value of the optimized objective for Problem 1 with $U^*(K_0)$.

Proposition 1. *An allocation* $\{e^*, K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ *that solves Problem* 1 *with* $e^* = 1$ *satisfies:*

$$\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} = \frac{\left[1 + \mu \frac{(\pi(1) - \pi(0))}{\pi(1)}\right]}{\left[1 - \mu \frac{(\pi(1) - \pi(0))}{(1 - \pi(1))}\right]} > 1, \tag{4}$$

$$u'(c_0^*) E_1 \left[\frac{1}{u'(c_1^*(x))}\right] = \beta E_1(1+x), \tag{5}$$

where $\mu > 0$ *is the multiplier on the incentive compatibility constraint (3).*

Proof. Letting μ be the multiplier on the incentive compatibility constraint and λ the one on the resource constraint, the first order necessary conditions for the planning problem at $e = 1$ are:

$$-u'(K_0 - K_1) + \lambda E_1(1+x) = 0,$$

$$(1 - \pi(1)) \beta u'(c_1(\underline{x})) - \mu(\pi(1) - \pi(0)) \beta u'(c_1(\underline{x})) - \lambda(1 - \pi(1)) = 0,$$

$$\pi(1)\,\beta u'(c_1(\bar{x})) - \mu(\pi(0) - \pi(1))\,\beta u'(c_1(\bar{x})) - \lambda\pi(1) = 0.$$

At $e = 0$, the same first order necessary conditions hold with $\mu = 0$. If $e^* = 1$ is optimal, the first order conditions can be simplified to yield (4) and (5). ■

Equation (4) implies that $c_1^*(\bar{x}) > c_1^*(\underline{x})-$ there is *partial insurance*. Equation (5) determines the intertemporal profile of constrained-efficient consumption. Equation (5) immediately implies:

$$u'(c_0^*) < \beta E_1(1+x)\,E_1\left[u'(c_1^*(x))\right],$$

by Jensen's inequality. Hence, there is a wedge between the entrepreneurs' intertemporal marginal rate of substitution and the aggregate intertemporal rate of transformation, which corresponds to $E_1(1+x)$. Using the first order necessary conditions for the planner's problem, this intertemporal wedge can be written as:

$$\begin{aligned} \text{IW} &= \beta E_1(1+x)\,E_1 u'(c_1^*(x)) - u'(c_0^*) \\ &= \beta E_1(1+x)\,\mu(\pi(1)-\pi(0))\left[u'(c_1^*(\underline{x})) - u'(c_1^*(\bar{x}))\right] > 0. \end{aligned} \tag{6}$$

The presence of an intertemporal wedge in dynamic economies with private information stems from the influence of outstanding wealth on the agent's attitude towards the risky distribution of outcomes in subsequent periods, which in turn affects incentives. The intertemporal wedge is a measure of the incentive cost of transferring wealth to a future period. In standard repeated moral hazard models, such as Rogerson (1985), higher wealth always has an adverse effect on incentives, because it reduces the dependence of consumption on the realization of uncertainty, and therefore on effort. Hence, the intertemporal wedge on assets with return equal to the aggregate intertemporal rate of transformation is positive.

In this economy, however, entrepreneurial capital is agent specific and associated with idiosyncratic risk in returns. Hence, the individual intertemporal rate of transformation is given by the stochastic variable $1 + x$, and does not correspond to $E_1(1+x)$. It is then useful to introduce the notion of an *individual* intertemporal wedge on entrepreneurial capital, and compare it to the *aggregate* intertemporal wedge defined in (6).

The individual intertemporal wedge is defined as the difference between the expected discounted value of idiosyncratic capital returns and the marginal utility of current consumption:

$$\text{IW}_K = \beta E_1 u'(c_1^*(x))(1+x) - u'(c_0^*). \tag{7}$$

By (5) and the definition of covariance, it immediately follows that:

$$\begin{aligned} \text{IW}_K &= \beta E_1 u'(c_1^*(x))(1+x) - u'(c_0^*) \\ &= \text{IW} + \beta Cov_1\left(u'(c_1^*(x)), x\right). \end{aligned}$$

Equation (4) and strict concavity of utility imply: $Cov_1 \left(u' \left(c_1^* \left(x\right)\right), x\right) < 0$. It follows that $\text{IW}_K <$IW and that the sign of IW_K can be *positive or negative*. This can also be seen by deriving IW_K from the first order necessary conditions for Problem 1:

$$\text{IW}_K = \mu \left(\pi \left(1\right) - \pi \left(0\right)\right) \beta \left[u' \left(c_1^* \left(\underline{x}\right)\right) \left(1 + \underline{x}\right) - u' \left(c_1^* \left(\bar{x}\right)\right) \left(1 + \bar{x}\right)\right]. \tag{8}$$

This expression clearly illustrates that the negative covariance between x and $u' \left(c_1^* \left(x\right)\right)$ could determine a negative sign for IW_K.

The possibility of a negative individual intertemporal wedge stems from the positive dependence of expected capital returns on entrepreneurial effort, which implies that more capital need not have an adverse effect on incentives. We now examine the incentive effects of capital in more detail.

2.1. The Effects of Capital and Assets on Entrepreneurial Incentives

To relate the sign of the intertemporal wedge to the effect of capital on entrepreneurial incentives, we consider the problem of an entrepreneur who maximizes lifetime utility by choice of effort and investment:

$$\left\{\hat{e}, \hat{K}_1\right\} = \arg \max_{K_1 \in [0, K_0],\ e \in \{0,1\}} U \left(e, K_1\right) - v \left(e\right),$$

where

$$U \left(e, K_1\right) \equiv u \left(K_0 - K_1\right) + \pi \left(e\right) u \left(K_1 \left(1 + \bar{x}\right)\right) + \left(1 - \pi \left(e\right)\right) u \left(K_1 \left(1 + \underline{x}\right)\right).$$

The Euler equation for this problem is:

$$U_{K_1} = -u' \left(K_0 - \hat{K}_1\right) + E_{\hat{e}} u' \left(\hat{K}_1 \left(1 + x\right)\right) \left(1 + x\right) = 0. \tag{9}$$

Equation (9) uncovers a complementarity between capital and entrepreneurial effort that drives the incentive effects of capital and is linked to the sign of the intertemporal wedge. This can be seen by totally differentiating (9), to yield:

$$\frac{\Delta e}{\Delta K_1} = \frac{-U_{K_1 K_1}}{\frac{\Delta U_{K_1}}{\Delta e}},$$

where:

$$\frac{\Delta U_{K_1}}{\Delta e} \equiv \left(\pi \left(1\right) - \pi \left(0\right)\right) \left[u' \left(\hat{c}_1 \left(\bar{x}\right)\right) \left(1 + \bar{x}\right) - u' \left(\hat{c}_1 \left(\underline{x}\right)\right) \left(1 + \underline{x}\right)\right], \tag{10}$$

is the discrete analogue of the off-diagonal term of the Hessian matrix in the agent's lifetime decision problem. By the concavity of u, the expressions $\frac{\Delta e}{\Delta K_1}$ and $\frac{\Delta U_{K_1}}{\Delta e}$ have the same sign.

Equation (10) can be rewritten as:

$$\frac{\Delta U_{K_1}}{\Delta e} \equiv \left(\pi\left(1\right)-\pi\left(0\right)\right)\left\{\left(\bar{x}-\underline{x}\right)u'\left(\hat{c}_1\left(\bar{x}\right)\right)+\left(1+\underline{x}\right)\left[u'\left(\hat{c}_1\left(\bar{x}\right)\right)-u'\left(\hat{c}_1\left(\underline{x}\right)\right)\right]\right\}. \quad (11)$$

This expression illustrates that the positive dependence of capital returns on effort generates a substitution effect, which corresponds to the first term inside the curly brackets, and an opposing wealth effect. The substitution effect tends to increase effort at higher levels of capital and its size is positively related to the spread in capital returns across states. The wealth effect tends to reduce effort for higher holdings of capital, since more capital increases consumption in the bad state. The size of the wealth effect is positively related to the spread in consumption across states, given that it is driven by the entrepreneurs' demand for insurance.[4]

The presence of two opposing forces in the relation between capital and effort determines the possibility of a negative intertemporal wedge. This can be seen by evaluating (10) at the constrained-efficient allocation. Then, by (8):

$$sign\left\{\frac{\Delta e}{\Delta K_1}\left(e^*,K_1^*\right)\right\} = sign\left\{\frac{\Delta U_{K_1}}{\Delta e}\left(e^*,K_1^*\right)\right\} = sign\left\{-\mathrm{IW}_K\right\}. \quad (12)$$

Hence, (12) implies that the individual intertemporal wedge is positive/negative when more capital tightens/relaxes the incentive compatibility constraint, given that if $\frac{\Delta U_{K_1}}{\Delta e}\left(e^*,K_1^*\right) \gtrless 0$, an entrepreneur will find it optimal to reduce/increase investment if she lowers her effort. In addition, by (11), the individual intertemporal wedge will be negative if the spread in capital returns is sufficiently large and/or the spread in consumption across states is sufficiently small.

By contrast, if entrepreneurs can also hold a riskless asset, B_1, with gross return $E_1\left(1+x\right)$, the corresponding Euler equation is:

$$U_{B_1} = -u'\left(\hat{c}_0\right)+E_1\left(1+x\right)E_{\hat{e}}u'\left(\hat{c}_1\right)=0. \quad (13)$$

The analogue of (10) for this asset is:

$$\frac{\Delta U_{B_1}}{\Delta e} \equiv \left(\pi\left(1\right)-\pi\left(0\right)\right)\left[u'\left(\hat{c}_1\left(\bar{x}\right)\right)-u'\left(\hat{c}_1\left(\underline{x}\right)\right)\right]E_1\left(1+x\right). \quad (14)$$

This expression is negative as long as $\hat{c}_1\left(\bar{x}\right) > \hat{c}_1\left(\underline{x}\right)$. It follows that an agent choosing $e=0$ will always choose a higher value of a riskless asset with return $E_1\left(1+x\right)$ relative to an agent choosing $e=1$. Hence, higher holdings of such an asset always tighten the incentive compatibility constraint. This negative complementarity between a riskless

[4] Levhari and Srinivasan (1969) and Sandmo (1970) study precautionary holdings of risky assets and discuss similar effects. See also Gollier (2001).

asset and effort is reflected in the positive intertemporal wedge, since when (14) is evaluated at the constrained-efficient allocation, it is proportional to $-\mathrm{IW}$.

The differential incentive effects of riskless assets and of entrepreneurial capital explain the fact that the individual intertemporal wedge on capital is always smaller than the wedge on a riskless asset with the same expected rate of return, and lead to a prescription of optimal differential taxation of these assets, which will we explore in section 3.1.

2.2. CRRA Examples

Restricting attention to utility functions in the CRRA class, with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, for $\sigma > 0$, we can further characterize the constrained-efficient allocation. Equation (8) implies:

$$
\begin{aligned}
& \mathrm{IW}_K K_1^{*} \sim \left[u'\left(c_1^*\left(\underline{x}\right)\right)\left(1+\underline{x}\right) - u'\left(c_1^*\left(\bar{x}\right)\right)\left(1+\bar{x}\right)\right] K_1^* \qquad (15) \\
< \; & \left[u'\left(c_1^*\left(\underline{x}\right)\right) c_1^*\left(\underline{x}\right) - u'\left(c_1^*\left(\bar{x}\right)\right) c_1^*\left(\bar{x}\right)\right] \\
= \; & \left(1-\sigma\right)\left[u\left(c_1^*\left(\underline{x}\right)\right) - u\left(c_1^*\left(\bar{x}\right)\right)\right],
\end{aligned}
$$

if $\left(1+\bar{x}\right) K_1^* > c_1^*\left(\bar{x}\right)$, and $c_1^*\left(\underline{x}\right) > \left(1+\underline{x}\right) K_1^*$, since $\frac{u'(c)c}{u(c)} = 1-\sigma$. Since $c_1^*\left(\underline{x}\right) < c_1^*\left(\bar{x}\right)$, it follows that if $1 > \sigma$, the intertemporal wedge will be negative. However, this condition does not restrict the sign of the intertemporal wedge for $\sigma \geq 1$, which is the empirical relevant case.

Since at the constrained-efficient allocation the spread in consumption across states is decreasing in the entrepreneurs' risk-aversion[5], the intertemporal wedge will be negative at sufficiently high levels of risk-aversion. For intermediate values of risk-aversion, the intertemporal wedge on capital will be negative for sufficiently large spread in capital returns.

To investigate the properties of optimal allocations in more detail, we now turn to numerical examples. We assume $v\left(e\right) = \gamma e^{1/\gamma}$, $\gamma > 0$, and $\pi\left(e\right) = a + be$, with $a \geq 0$, $b > 0$ and $2a + b \leq 1$. The parameter b represents the impact of effort on capital returns. We set $a = 0$, so that at low effort capital is risk free, and $b = 0.5$. We interpret x as percentage earnings on entrepreneurial capital, which we identify as private equity. We parameterize the distribution of x with the distribution of earnings conditional on survival for private equity in Moscowitz and Vissing-Jorgensen (2002). This corresponds to $\{\underline{x}, \bar{x}\} = \{0.3, 0.7\}$, which implies $E_1 x = 0.5$, $SD_1\left(x\right) = 0.2$, $E_0\left(x\right) = 0.3$, where SD_e denotes the standard deviation, conditional on effort e.[6] We fix $\gamma = 0.1111$. We consider

[5] This property always holds in numerical simulations.

[6] The average returns to private equity, including capital gains and earnings, are estimated from SCF data to be 12.3, 17.0 and 22.2 percent per year in the time periods 1990-1992, 1993-1995, 1996-1998, as reported in Moskowitz and Vissing-Jorgensen (2002).

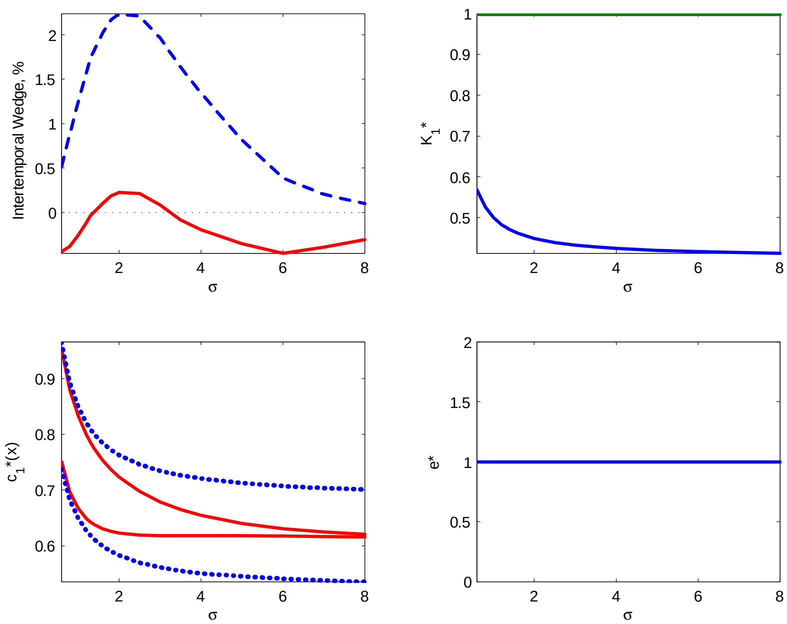


Figure 1: Benchmark parameters

several other parameterizations of the capital returns distribution to check robustness. We set $K_0 = 1$ for all the parameterizations considered.

We first compute the optimal allocation and the intertemporal wedge as a function of σ. Our findings for the benchmark parameterization are displayed in figure 1. The individual intertemporal wedge on K_1 is the solid line, while the dashed line corresponds to the aggregate intertemporal wedge. The individual intertemporal wedge is negative for low and high values of σ, while it is positive for intermediate values of σ. The aggregate intertemporal wedge is always positive. Investment is decreasing in σ. In the third panel, we plot constrained-efficient consumption (solid line) and total capital earnings, $K_1^* (1 + x)$ (dotted line), in each state. The spread in optimal consumption across states decreases with σ, for given spread in capital returns. This contributes to a negative value of the intertemporal wedge as σ increases.

Figure 2 shows the properties of the optimal allocation for a smaller spread in capital returns, with $\{\underline{x}, \bar{x}\} = \{0.25, 0.75\}$ which corresponds to $E_1 x = 0.5$, $E_0 x = 0.25$, $SD_1 (x) = 0.25$. From (8), we know that a larger spread in capital returns increases the value of the individual intertemporal wedge on capital. For this parameterization, we find that the intertemporal wedge on capital is always negative. The qualitative properties of the constrained-efficient allocation as a function of σ are similar to the

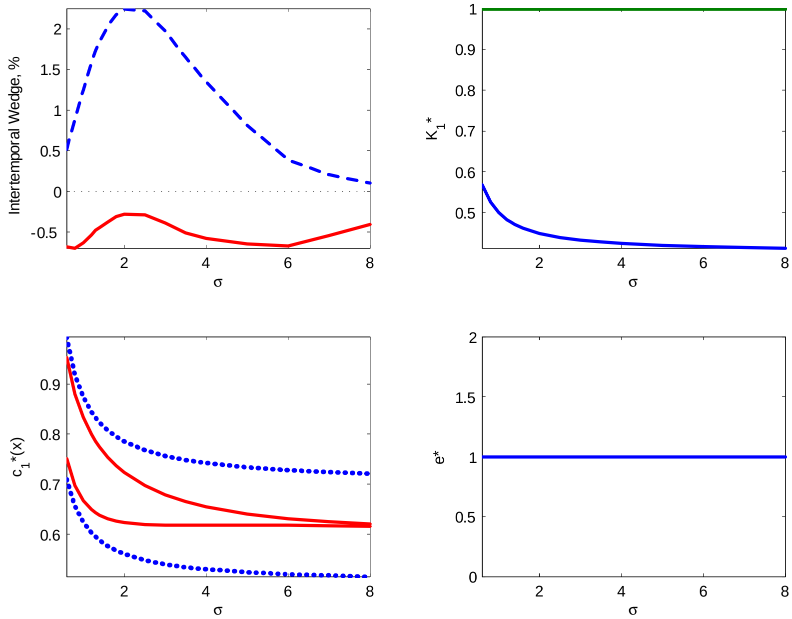


Figure 2: Benchmark parameterization

previous example.

To further investigate the effect of the spread in capital returns on the sign of the intertemporal wedge, we set σ equal to 1.5 and vary $(\bar{x} - \underline{x})$, so that the standard deviation of capital returns ranges between 12.5 and 27.5 percent, while expected capital returns are constant. All other parameters are as in the previous example. The results are displayed in figure 3, The optimal allocation only depends on expected capital returns and does not vary with the spread in capital returns. The individual intertemporal wedge on capital is decreasing in the spread in capital returns and eventually turns negative.

3. Optimal Taxes

We now consider how to implement constrained-efficient allocations in a setting where agents can trade in competitive markets. We explore different market structures. A market structure specifies the distribution of ownership rights, the feasible trades between agents and the information structure. Agents are subject to taxes that influence their budget constraints. A tax system implements the constrained-efficient allocation if such an allocation arises as the competitive equilibrium outcome under this tax system

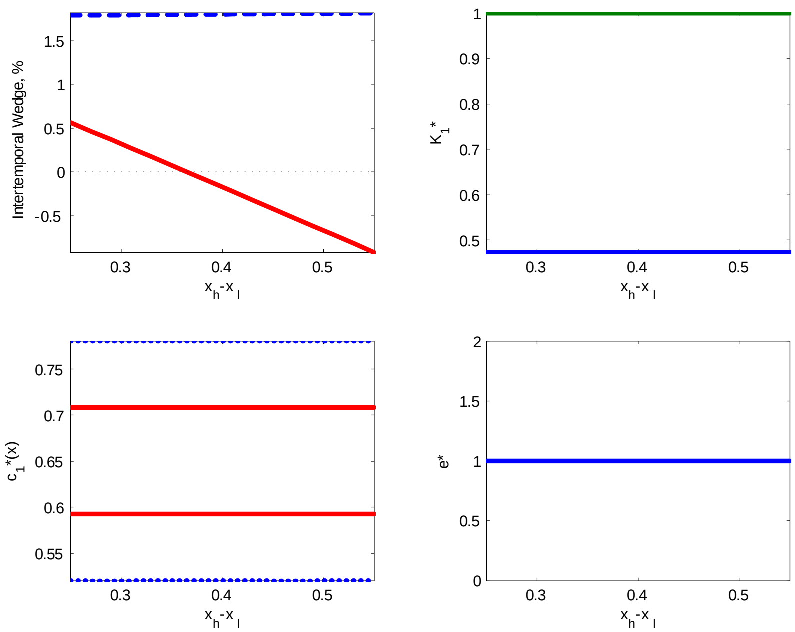


Figure 3: Constrained-efficient allocations and the spread in capital returns.

for a particular market structure. We identify the optimal tax system as the one that implements the constrained-efficient allocation. The only ex ante constraint imposed on the tax system is that it must specify transfers that are conditioned only on agents' observable characteristics.

3.1. Optimal Differential Asset Taxation

The first market structure we consider is one in which agents can trade risk-free bonds and independently choose investment as well as effort at time 0. The risk-free bonds yield a return r in period 1, which is determined in equilibrium. Decisions occur as follows. Agents are endowed with initial capital K_0 and choose K_1 and bond purchases B_1 at the beginning of period 0, and they consume. They then exert effort. At the beginning of period 1, x is realized. Then, the government collects taxes and agents consume. The informational structure is as follows: K_1 and x are public information, while effort is private information. We also assume that bond purchases B_1 are public information. The tax system is given by a time 1 transfer from the agents to the government which is conditional on observables and represented by the function $T(B_1, K_1, x)$. We restrict attention to functions T that are differentiable almost everywhere in their first argument and satisfy $E_1 T(B_1, K_1, x) = 0$, which corresponds to the government budget constraint, given that the government does not have any spending requirements.

An entrepreneur's problem is:

$$\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0, K_0, T) = \arg \max_{K_1 \in [0, K_0],\ B_1 \geq \bar{B},\ e \in \{0,1\}} U(e, K_1, B_1; T) - v(e), \qquad \text{(Problem 3)}$$

where

$$U(e, K_1, B_1; T) = u(K_0 + B_0 - K_1 - B_1) + E_e u(K_1(1+x) + B_1(1+r) - T(K_1, B_1, x)),$$

subject to $K_0 + B_0 - K_1 - B_1 \geq 0$ and $K_1(1+x) + (1+r)B_1 - T(B_1, K_1, x) \geq 0$ for $x \in X$. Here, the debt limit $\bar{B}$ is imposed to ensure that an agent's problem is well defined. The natural debt limit for tax systems in the class $T(B_1, K_1, x) = \rho(x) + \tau_B(x) B_1 + \tau_K(x) K_1$ is $\bar{B} = -\frac{[K_1(1+\underline{x}-\tau_K(\underline{x}))-\rho(\underline{x})]}{1+r-\tau_B(\underline{x})}$. This limit ensures that agents will be able to pay back all outstanding debt in the low state.

We interpret the initial bond endowment, B_0, as a transfer from the government to the entrepreneurs. Consequently, we allow the government to issue bonds at time 0, denoted B_1^G. The government budget constraints at time 0 and at time 1 are, respectively, $B_0 - B_1^G \leq 0$ and $E_e T(K_1, B_1, x) - B_1^G(1+r) \geq 0$, where e corresponds to the effort chosen by the entrepreneurs.

Definition 2. *A competitive equilibrium is an allocation* $\{c_0, e, K_1, B_1, c_1(\underline{x}), c_1(\bar{x})\}$ *and initial endowments* B_0 *and* K_0 *for the entrepreneurs, a tax system* $T(K_1, B_1, x)$, *with* $T : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \to \mathbb{R}$, *government bonds* B_1^G, *and an interest rate,* $r \geq 0$, *such that: i) given* T *and* r *and the initial endowments, the allocation solves Problem 3; ii) the government budget constraint holds in each period; iii) the bond market clears,* $B_1^G = B_1$.

The restriction on the domain of the tax system is imposed to ensure that the tax is specified for all values of K_1 and B_1 feasible for the entrepreneurs. Since entrepreneurs are all ex ante identical, if the government does not issue any bonds, $B_1 = 0$ in any competitive equilibrium. We allow the government to issue bonds to extend the analysis to the case in which bond holdings are in positive net supply. Given that the government does not need to finance any expenditures, the amount of government bonds issued does not influence equilibrium consumption, capital and effort allocations, or the equilibrium interest rate. However, if the government did have an expenditure stream to finance, the choice of bond holdings would be consequential. We now define our notion of implementation.

Definition 3. *A tax system* $T : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \to \mathbb{R}$ *implements the constrained-efficient allocation, if the allocation* $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$, *the tax systemT, jointly with an interest rate* r, *government bonds* B_1^G, *and initial endowments* B_0 *and* K_0 *constitute a competitive equilibrium.*

We restrict attention to separable tax systems of the form: $T(K_1, B_1, x) = \rho(x) + \tau_K(x) K_1 + \tau_B(x) B_1$. Let $B_1^* \geq \bar{B}$ a level of bond holdings to be implemented. If B_0 and $T(K_1^*, B_1^*, x)$ respectively satisfy:

$$c_0^* = B_0 + K_0 - K_1^* - B_1^*, \tag{16}$$

$$c_1^*(x) = K_1^*(1 + x) + (1 + r) B_1^* - T(K_1^*, B_1^*, x). \tag{17}$$

then, K_1^* and B_1^* are affordable and, if they are chosen by an entrepreneur, incentive compatibility implies that high effort will also be chosen. Evaluating the Euler equation at $\{1, K_1^*, B_1^*\}$, we can write:

$$u'(c_0^*) = \beta E_1 \left[u'(c_1^*(x)) (1 + x - \tau_K(x)) \right], \tag{18}$$

$$u'(c_0^*) = \beta E_1 \left[u'(c_1^*(x)) (1 + r - \tau_B(x)) \right]. \tag{19}$$

The restrictions on $T(K_1^*, B_1^*, x)$ implied by (16)-(17) and (18)-(19) do not fully pin down the properties of the tax system and do not ensure that the constrained-efficient allocation is chosen by an entrepreneur. To see this, let $\tau_K(\bar{x}) = \tau_K(\underline{x}) = \bar{\tau}_K$ and $\tau_B(\bar{x}) = \tau_B(\underline{x}) = \bar{\tau}_B$, so that marginal asset taxes do not depend on x, with $\bar{\tau}_K$ and $\bar{\tau}_B$ that satisfy (18)-(19). Then, $\bar{\tau}_K$ has the same sign as the intertemporal wedge on capital, while $\bar{\tau}_B$ is always positive, since the intertemporal wedge on the bond is positive. Set

$\bar{\rho}(x)$ so that (17) holds under $\bar{\tau}_K$, $\bar{\tau}_B$, and let $\bar{T}(K_1, B_1, x) = \bar{\rho}(x) + \bar{\tau}_K K_1 + \bar{\tau}_B B_1$. It follows that:

$$u'(c_0^*) \lesseqgtr \beta E_0 \left[u'(c_1^*(x)) (1 + x - \bar{\tau}_K) \right] \text{ if } \mathrm{IW}_K \gtreqless 0, \tag{20}$$

$$u'(c_0^*) < \beta (1 + r - \bar{\tau}_B) E_0 u'(c_1^*(x)). \tag{21}$$

Since the incentive compatibility constraint is binding, this implies that entrepreneurs would find it optimal to increase holdings of bonds and increase/reduce holdings of capital if the intertemporal wedge on capital is positive/negative and exert low effort, rather than choose $\{1, K_1^*, B_1^*\}$ which is affordable and satisfies first order necessary conditions. Hence, the tax system $\bar{T}(K_1, B_1, x)$ does not implement the constrained-efficient allocation.[7]

We now construct a tax system that implements the constrained-efficient allocation. The critical property of this system is that it involves marginal asset taxes that depend on observable capital returns.

Proposition 4. *A tax system $T^*(B_1, K_1, x) = \rho^*(x) + \tau_B^*(x) B_1 + \tau_K^*(x) K_1$, with $T^* : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \to \mathbb{R}$, and an initial bond endowment B_0^* that satisfy:*

$$1 + r - \tau_B^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{22}$$

$$1 + x - \tau_K^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{23}$$

$$c_1^*(x) = K_1^*(1 + x - \tau_K^*(x)) + B_1^*(1 + r - \tau_B^*(x)) - \rho^*(x), \tag{24}$$

and

$$c_0^* = B_0^* + K_0 - K_1^* - B_1^*, \tag{25}$$

ensure that the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ is optimal for entrepreneurs for some $B_1^* \geq \bar{B}$ and some $r \geq 0$.

Proof. We want to show that

$$\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0^*, K_0, T^*) = (1, K_1^*, B_1^*),$$

for some $B_1^* \geq \bar{B}$ and for given r. Suppose that $\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0^*, K_0, T^*) \neq (1, K_1^*, B_1^*)$. If $\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}$ is interior in $\hat{K}_1$ and $\hat{B}_1$, at T^* :

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}.$$

[7] The result that non-state dependent marginal asset taxes allow for devations from the constrained-efficient allocation also holds in economies with idiosyncratic labor risk, as discussed in Albanesi and Sleet (2006) and Kocherlakota (2005). Golosov and Tsyvinski (2006) derive a related result in a disability insurance model.

It follows that for any interior $\hat{K}_1, \hat{B}_1$ such that $\hat{K}_1 + \hat{B}_1 \gtrless K_1^* + B_1^*$, then (22) and (23) imply $\frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \lessgtr \frac{u'\left(c_0^*\right)}{u'\left(c_1^*(x)\right)}$ irrespective of the value of $\hat{e}$, a contradiction. Moreover, at T^*, the local sufficient conditions for optimality are also satisfied irrespective of the value of $\hat{e}$. To see this, consider the sub-optimization problem associated with the choice of B_1 and K_1 for given e. The elements of the Hessian, H_U, for this problem are:

$$U_{BB}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + r - \tau_B^*(x))^2 \leq 0,$$

$$U_{KK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + x - \tau_K^*(x))^2 \leq 0,$$

$$U_{BK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + r - \tau_B^*(x))(1 + x - \tau_K^*(x)),$$

where U_{xy} denotes a cross-partial derivative with respect to the variables x, y.Under (22)-(23), $|H_U| = 0$. Hence, the Hessian is negative semi-definite irrespective of the value of $\hat{e}$. We now consider values of $\hat{K}_1, \hat{B}_1$ that are not interior. The Inada conditions exclude non-interior solutions that result from the non-negativity constraint on time 0 consumption being binding. Hence, there are two candidate non-interior solutions: $\hat{K}_1 = 0$ and $\hat{B}_1 > 0$, and $\hat{K}_1 > 0$ and $\hat{B}_1 = \bar{B}$. In both cases, one of the Euler equations must hold with equality and the other as a strict inequality. Under T^*:

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}, \tag{26}$$

$$1 > E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}. \tag{27}$$

This is a contradiction, since (26) and (27) clearly cannot hold at the same time. Then, K_1^*, B_1^* are globally optimal irrespective of the value of $\hat{e}$. At at K_1^*, B_1^*, $\rho^*(x)$ implies $\hat{e} = 1$ since the constrained-efficient allocation is incentive compatible. Hence, the allocation $\{1, K_1^*, B_1^*\}$ is optimal for the agent given the initial endowments B_0^*, K_0, the tax system T^*, and the interest rate r. ■

The tax system T^* removes the complementarity between effort and investment and effort and bond holdings and thus it guarantees that the necessary and sufficient conditions for the *joint* global optimality of K_1^* and B_1^* are satisfied at all effort levels. Moreover, T^* equates after tax returns on all assets in each state. This makes entrepreneurs' indifferent over the composition of their portfolio. We now prove that the tax system T^* implements the constrained-efficient allocation.

Corollary 5. *The tax system $T^*(K_1, B_1, x)$ and initial bond endowment B_0^* defined in proposition 4, jointly with the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$, and government bonds B_1^G, with $B_0^* = B_1^* = B_1^G \geq \bar{B}$, a return r, constitute a competitive equilibrium for the market economy with initial capital K_0.*

Proof. By Proposition 4, for any $r \geq 0$ and $B_1^* \geq \bar{B}$, the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ solves the agents' optimization problem in the market economy for initial endowments B_0^* and K_0. In addition, at $B_0^* = B_1^* = B_1^G$ the bond market clears and the resource constraint is satisfied at time 0. The resource constraint at time 1 is satisfied by construction. Hence, by (24), $E_1 c_1^*(x) = K_1 E_1 (1+x) + B_1^* (1+r) - E_1 T^* (K_1^*, B_1^*, x)$, so that the government budget constraint is satisfied at time 1. ∎

The following corollary characterizes the properties of the optimal tax system. The average marginal capital tax is zero. The marginal capital tax is decreasing in capital returns, and thus appears regressive, if the intertemporal wedge is positive, while it is increasing in capital returns and thus appears regressive is the intertemporal wedge is negative. The average marginal tax on bonds is also equal to 0, while marginal bond taxes are decreasing in income and thus appear regressive. Capital is subsidized with respect to bonds in the bad state.

Proposition 6. *The tax system $T^*(B_1, K_1, x)$ defined in proposition 4 implies:*
i) $E_1 \tau_K^*(x) = 0$;
ii) $sign(\tau_K^*(\bar{x}) - \tau_K^*(\underline{x})) = sign(-IW_K)$;
iii)$E_1 \tau_B^*(x) = r - E_1(x)$;
iv) $\tau_B^*(\bar{x}) < \tau_B^*(\underline{x})$;
v) $\tau_B^*(\underline{x}) > \tau_K^*(\underline{x})$ *and* $\tau_B^*(\bar{x}) < \tau_K^*(\bar{x})$;
vi) the intertemporal wedge associated with the risk-less bond is greater than the intertemporal wedge associated with risky productive capital.

Proof. By (22):

$$E_1 \left[1 + x - \frac{u'(c_0^*)}{u'(c_1^*(x))} \right] = E_1 \tau_K^*(x),$$

which from (5) implies i). (22) also implies:

$$u'(c_1^*(\overline{x}))\tau_K^*(\overline{x}) - u'(c_1^*(\underline{x}))\tau_K^*(\underline{x}) = u'(c_1^*(\overline{x}))(1+\overline{x}) - u'(c_1^*(\underline{x}))(1+\underline{x}).$$

Since:

$$sign\left[u'(c_1^*(\overline{x}))(1+\overline{x}) - u'(c_1^*(\underline{x}))(1+\underline{x})\right] = sign\,(\text{-IW}_K)$$

and $u'(c_1^*(\overline{x})) < u'(c_1^*(\underline{x}))$, ii) follows. iii) follows from the planner's Euler equation, since:

$$E_1 \tau_B^*(x) = 1 + r - E_1 \left(\frac{u'(c_0^*)}{\beta u'(c_1^*(x))} \right).$$

iv) follows directly from (22) and $u'(c_1^*(\bar{x})) < u'(c_1^*(\underline{x}))$. To show v) note that (22) and (23) imply $\tau_B^*(x) - \tau_K^*(x) = E_1 x - x$. The intertemporal wedge associated with each asset is:

$$\begin{aligned} E_1 u'\left(c_1^*(x)\right)(1+x) - u'\left(c_0^*\right) &= E_1 u'\left(c_1^*(x)\right)\tau_K^*(x), \\ E_1 u'\left(c_1^*(x)\right) E_1(1+x) - u'\left(c_0^*\right) &= E_1 u'\left(c_1^*(x)\right)\tau_B^*(x). \end{aligned}$$

vi) follows immediately. ■

The statement of corollary 5 and result iii) in proposition 6 illustrate that the equilibrium values of r and $E_1\tau_B^*(x)$ are not separately pinned down. Any value of r and $E_1\tau_B^*(x)$ that satisfy $E_1\tau_B^*(x) = r - E_1(x)$ is consistent with the entrepreneurs' Euler equations and all other equilibrium conditions. This indeterminacy does not affect the dependence of marginal bond taxes on x, which is pinned down by (23). Hence, without loss of generality we restrict attention to competitive equilibria with $r = E_1(x)$ and $E_1\tau_B^*(x) = 0$.

The optimal marginal capital and bond taxes for the benchmark parameterization discussed in section 2.2 are plotted in figure 4, for $r = E_1(x)$. The solid line in each panel corresponds to the intertemporal wedge. The dashed-asterix line corresponds to marginal taxes in state $\underline{x}$, whereas the dashed-cross line corresponds to optimal marginal taxes in state $\bar{x}$. The marginal tax on capital, displayed in the left panel, is negative in the low state and positive in the good state, while the opposite is true for the marginal tax on bonds. Hence, the marginal capital tax is increasing in earnings, while the marginal bond tax is decreasing in earnings. Despite the fact that wedges are quite small in percentage terms, the magnitude of marginal taxes is significant. The capital tax ranges from 5 to 19% in absolute value as a function of σ, while the bond tax ranges from 6 to 30% in absolute value.

In figure 3.1, we report optimal marginal capital and bond taxes as a function of the spread in capital returns. This corresponds to the constrained-efficient allocation in figure 3. Since the constrained-efficient allocation only depends on the expected value of capital returns and not on their spread, the marginal bond tax taxes are constant. Instead, as discussed, the intertemporal wedge on capital is decreasing in the spread of capital returns. When IW_K is positive, the marginal capital tax is decreasing in capital returns, but the marginal tax on capital in the bad state is always lower than the marginal tax on bonds in the bad state.

The main finding in the fiscal implementation for the market structure with riskless bonds is the *optimality of differential asset taxation.* There are two aspects of this result. First, the intertemporal wedge on a riskless asset is always greater than the intertemporal wedge on entrepreneurial capital. Second, entrepreneurial capital should be subsidized relatively to a riskless asset in the bad state, irrespective of the sign of the intertemporal wedge. The optimal tax system equalizes after tax returns on entrepreneurial capital and riskless bonds, thus it reduces the after tax spread in capital returns and it increases the after tax spread in the returns to the riskless bond.

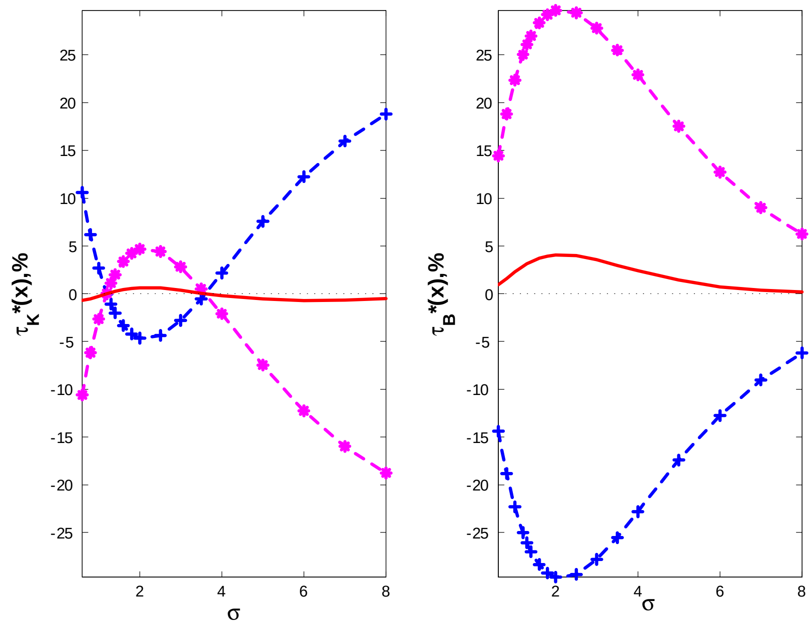
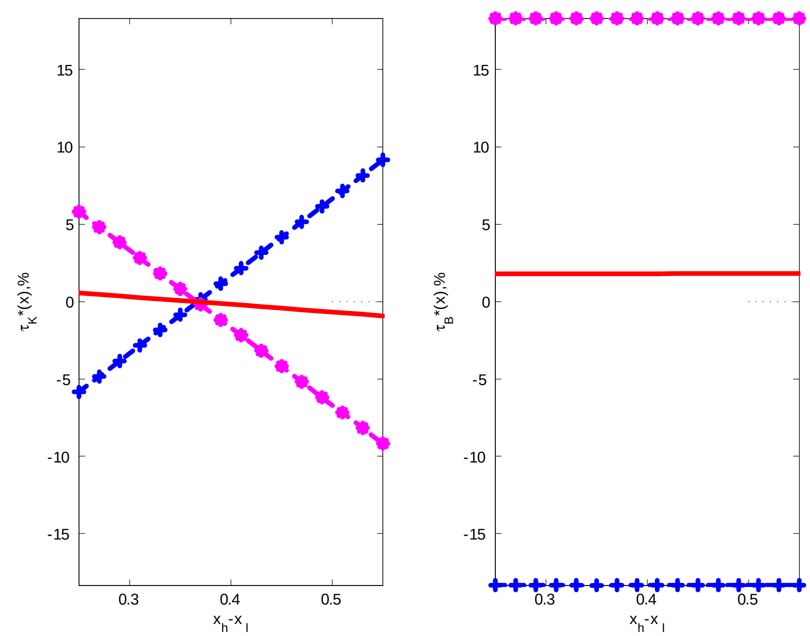


Figure 4: Optimal capital and bond taxes, larger cost of effort.



These results can be generalized to risky securities. Consider a security with return $r(x) > 0$ for $x = \underline{x}, \bar{x}$, in zero net supply. Assume that entrepreneurs can trade this security at price q at time 0. Letting the candidate tax system be given by $T(S_1, K_1, x) = \tau_K(x) K_1 + \tau_S(x) S_1 + \rho(x)$ .Set $\tau_K^*(x)$ and $\rho^*(x)$ as in (23) and (24) for $S_1^* = 0$. Set marginal taxes on the security according to:

$$1 + r(x) - \tau_S^*(x) = \frac{qu'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{28}$$

The equilibrium price of the security is $q = \frac{E_1\left(1+r(x)-\tau_S^*(x)\right)}{E_1\left(1+x-\tau_K^*(x)\right)}$[8]. Then, (28) implies $E_1\tilde{r}(x) = E_1 x$, where $\tilde{r}(x)$ is the equilibrium rate of return on this security, $\tilde{r}(x) = \frac{1+r(x)}{q} - 1$.

The intertemporal wedge on the risky security is:

$$\text{IW}_S = E_1 u'(c_1^*(x))(1 + \tilde{r}(x)) - u'(c_0^*),$$

Following the usual reasoning:

$$sign\{\text{IW}_S\} = sign\left\{u'(c_1^*(\underline{x}))(1 + \tilde{r}(\underline{x})) - u'(c_1^*(x))(1 + \tilde{r}(\bar{x}))\right\}.$$

Then, if $Cov_1(\tilde{r}(x), x) \leq 0$, the intertemporal wedge on the risky security is positive. The intertemporal wedge can be positive or negative if $Cov_1(\tilde{r}(x), x) > 0$. The following result holds.

Proposition 7. *If $Cov_1(\tilde{r}(x), x) > 0$ and $V_1(x) > V_1(\tilde{r}(x))$, then:*

$$E_1 u'(c_1^*(x))(1 + \tilde{r}(x)) > E_1 u'(c_1^*(x))(1 + x),$$

$$\tau_S^*(\bar{x}) - \tau_K^*(\bar{x}) < 0 \text{ and } \tau_S^*(\underline{x}) - \tau_K^*(\underline{x}) > 0.$$

Proof. This follows from:

$$\begin{aligned} E_1 u'(c_1^*(x))(1 + \tilde{r}(x)) - E_1 u'(c_1^*(x))(1 + x) &= Cov_1\left(u'(c_1^*(x)), \tilde{r}(x)\right) - Cov_1\left(u'(c_1^*(x)), x\right) \\ &= Cov_1\left(u'(c_1^*(x)), \tilde{r}(x) - x\right). \end{aligned}$$

$Cov_1(u'(c_1^*(x)), \tilde{r}(x) - x) > 0$ if $\tilde{r}(x) - x$ is decreasing in x, or $Cov_1(\tilde{r}(x) - x, x) < 0$. By the definition of covariance and by the fact that $E_1 x = E_1\tilde{r}(x)$:

$$Cov_1(\tilde{r}(x) - x, x) = E_1\tilde{r}(x)x - E_1 x^2 = Cov_1(\tilde{r}(x), x) - V_1(x). \tag{29}$$

[8] As in the case with risk-free bonds, the equilibrium expected return on this security is not separately pinned down from $E_1\tau_s^*(x)$.

By $V_1(x) > V_1(\tilde{r}(x))$ and $Cov_1(\tilde{r}(x), x) > 0$, $0 < Corr_1(\tilde{r}(x), x) < 1$. Then:

$$Cov_1(\tilde{r}(x), x) - V_1(x) = SD_1(x)\left[Corr_1(\tilde{r}(x), x) SD_1(\tilde{r}(x)) - SD_1(x)\right] < 0.$$

In addition, $\tau_S^*(x) - \tau_K^*(x) = \tilde{r}(x) - x$. Since $\tilde{r}(x) - x$ is decreasing in x and $E_1\tilde{r}(x) = E_1 x$, $\tau_S^*(\bar{x}) - \tau_K^*(\bar{x}) < 0$ and $\tau_S^*(\underline{x}) - \tau_K^*(\underline{x}) > 0$. ■

If $Cov_1(\tilde{r}(x), x) > 0$ and $V_1(x) > V_1(\tilde{r}(x))$, $Corr_1(\tilde{r}(x), x) \in (0, 1)$, where $Corr_e$ denotes the correlation conditional on $\pi(e)$. The proposition states that a security positively correlated with capital with lower variance of returns has a higher intertemporal wedge than capital. An entrepreneur would be willing to hold such a security instead of capital, since it is associated with lower earnings risk. However, this has an adverse effect on incentives. This motivates the higher intertemporal wedge and the fact that $\tau_S^*(x) - \tau_K^*(x)$ is decreasing in x, which implies that capital is subsidized with respect to the risky security in the bad state.

This finding points to a general principle. It is the correlation of an asset's returns with the idiosyncratic risk that determines the asset's effects on the entrepreneurs' incentives to exert effort and, consequently, the properties of optimal marginal taxes on the asset.

In this section, we considered risk-free bonds and other financial securities in zero net supply. In the next section, we consider an implementation in which entrepreneurs can sell shares of their own capital to external investors, thus giving rise to an equity market with a positive supply of securities.

3.2. Optimal Capital Taxation with External Ownership

We now allow entrepreneurs to sell shares of their capital and buy shares of other entrepreneurs' capital. Each entrepreneur can be interpreted as a firm, so that this arrangement introduces an equity market. The amount of capital invested by an entrepreneur can be interpreted as the size of their firm.

An entrepreneur's budget constraint in each period is :

$$c_0 = K_0 - K_1 - \int_{i \in [0,1]} S_1(i)\, di + sK_1, \tag{30}$$

$$c_1(x) = K_1(1 + x) - sK_1(1 + d(x)) + \int_{i \in [0,1]} (1 + D(i)) S_1(i)\, di - T(K_1, s, \{S_1\}_i, x), \tag{31}$$

where $s \in [0, 1]$ is the fraction of capital sold to outside investors, $d(x)$ denotes dividends distributed to shareholders, $S_1(i)$ is the value of shares in company i in an entrepreneur's portfolio and $D(i, \tilde{x})$ denotes dividends earned from each share of company i if the realized returns are $\tilde{x}$ for $\tilde{x} \in X$. The dividend distribution policy is taken as given by the entrepreneurs and the shareholders. This arrangement should be interpreted as part of the share issuing agreement. Gross stock earnings for an entrepreneur with equity portfolio $\{S_1(i)\}_i$ are given by $\int_{i\in[0,1]}(1+D(i))\,S_1(i)\,di$, where $D(i)$ denotes expected dividends from firm i[9]. Entrepreneurs choose K_1, $\{S_1(i)\}_i$ as well as effort at time 0, taking as given the distribution policy, dividends and taxes. At time 1, x is realized, dividends are distributed, the government collects taxes and the entrepreneurs consume.

We consider candidate tax systems of the form $T(K_1,\{S_1\}_i,x)=\tau_P(x)(1+x)K_1+\tau_s(x)\int_i S_1(i)\,di+\rho(x)$. Here, $\tau_P(x)$ can be interpreted as a marginal tax on gross profits. The marginal tax on stock returns, $\tau_S(x)$, is the same for all stocks. The arguments are variables that are assumed to be public information.

The entrepreneurs' problem is:

$$\left\{\hat{e},\hat{K}_1,\hat{s},\left\{\hat{S}_1(i)\right\}_i\right\}(K_0,T)=\arg\max_{\hat{e},\hat{K}_1,\hat{s},\left\{\hat{S}_1(i)\right\}_i} u(c_0)+E_e u(c_1)-v(e), \quad \text{(Problem 4)}$$

subject to (30), (31) and $\int_{i\in[0,1]}S_1(i)\,di\geq\bar{B}=\frac{K_1(1+\underline{x})(1-\tau_P(\underline{x}))-\rho(\underline{x})}{(1-\tau_S(\underline{x}))\int_{i\in[0,1]}(1+D(i))di}$, where $\bar{B}$ is the natural borrowing limit.

An entrepreneur's Euler equations are:

$$\begin{aligned}0 &= -(1-s)\left\{u'(c_0)-\beta E_{\hat{e}}\left[(1+x)(1-\tau_P(x))\,u'(c_1(x))\right]\right\} \\ &\quad +\beta s E_{\hat{e}}\left[(1+x)(1-\tau_P(x))-(1+d(x))\right]u'(c_1(x)),\end{aligned} \tag{32}$$

$$-u'(c_0)+\beta E_{\hat{e}}(1+D(i)-\tau_S(x))\,u'(c_1(x))=0, \tag{33}$$

$$\left[u'(c_0)-\beta E_{\hat{e}}(1+d(x))\,u'(c_1(x))\right]K_1\begin{cases}=0 \text{ for } s\in(0,1)\\ \leq 0 \text{ for } s=0\\ >0 \text{ for } s=1.\end{cases}. \tag{34}$$

We define a competitive equilibrium for this trading structure and then consider how to implement the constrained-efficient allocation.

Definition 8. *A competitive equilibrium is an allocation* $\left\{c_0,\ \hat{e},\ \hat{K}_1,\ \hat{s},\ \left\{\hat{S}_1(i)\right\}_i,\ \hat{c}_1(x)\right\}$ *with* $\hat{s}\in[0,1]$, *a distribution policy* $\hat{d}(x)$ *and a dividend process* $\hat{D}(i,x)$ *for* $i\in[0,1]$, $x\in X$, *and a tax system* $T(K_1,\{S_1\}_i,x)$, *such that:*

i) the allocation $\left\{\hat{e},\hat{K}_1,\hat{s},\left\{\hat{S}_1(i)\right\}_i\right\}$ *solves the entrepreneurs' problem, for given* $\hat{d}(x)$, $\hat{D}(i,\tilde{x})$, *and* T;

ii) the dividend process is consistent with the distribution policy, $\hat{d}(x)=\hat{D}(i,x)$ *for all* i *and* $x\in X$;

[9] This holds since x is i.i.d. across entrepreneurs and the law of large numbers hold.

ii) the stock market clears, $\hat{s}\hat{K}_1 = \hat{S}_1(i)$ *for* $i = [0,1]$;
iii) the resource constraint is satisfied in each period.

Since all entrepreneurs are ex ante identical, we restrict attention to symmetric equilibria in which s, K_1 and effort are constant for all entrepreneurs. Consequently, $D(i) = E_{\hat{e}} D(i, \tilde{x})$, and $D(i,x) = d(x)$ for all i. The entrepreneurs face a portfolio problem in the selection of stocks. Since all stocks have the same expected return net of taxes, entrepreneurs are indifferent over which stocks to hold. However, they will always hold a continuum of stocks, since this ensures that their portfolio has zero variance. To break the entrepreneurs' indifference over portfolio selection, we assume that all entrepreneurs hold a perfectly differentiated portfolio. Hence, we can restrict attention to the case $S_1(i) = S_1$ and $D(i) = D$ for all $i = [0,1]$.

We now construct a tax system that implements the constrained-efficient allocation. We set the marginal profit tax is $\tau_P^*(x)$ as follows:

$$(1+x)(1-\tau_P^*(x)) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{35}$$

We assume that dividends per share are simply given by after tax profits, so that $d^*(x) = (1+x)(1-\tau_P(x)) - 1$. This implies: $1 + D^* = E_1(1+x)(1-\tau_P^*(x))$. We then set $\tau_S^*(x)$ so that:

$$1 + D^* - \tau_S^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{36}$$

where $D^* = E_1 x - E_1 \tau_P^*(x) - E_1 \tau_P^*(x) x$.

Lastly, we choose $\rho^*(x)$ to satisfy:

$$\begin{aligned} c_1^*(x) &= K_1^*(1+x)(1-\tau_P^*(x)) - s^* K_1^*(1+d^*(x)) \\ &\quad + (1 + D^* - \tau_S^*(x)) S_1^* - \rho^*(x), \end{aligned} \tag{37}$$

for some $s^* \in [0,1)$, with $S_1^* = s^* K_1^*$.

Proposition 9. *The tax system* $T^*(K_1, \{S_1(i)\}_i, x) = \tau_P^*(x)(1+x)K_1 + \tau_s^*(x)\int_i S_1(i)\,di + \rho^*(x)$, *where* $\tau_P^*(x)$, $\tau_S^*(x)$ *and* $\rho^*(x)$ *satisfy (35), (36) and (37), respectively, implements the constrained-efficient allocation with distribution policy* $1 + d^*(x) = (1+x)(1-\tau_P^*(x))$ *and dividend process* $D^*(i)$ *for all* i*. The allocation* $\{K_1^*, s^*, \{S_1^*(i)\}_i, 1, c_1^*(x)\}$ *with* $s^* K_1^* = S_1^*(i)$ *for all* i *and* $s^* \in [0,1)$*, the tax system* $T^*(K_1, \{S_1(i)\}_i, x)$*, the distribution policy* $d^*(x)$ *and the dividend process* $D^*(i,x)$ *constitute a competitive equilibrium.*

Proof. Suppose that $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}(K_0, T) \neq \{1, K_1^*, s^*, \{s^* K_1^*\}_i\}$ for some $s^* \in [0,1)$. If $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}$ is interior, by (35), (36) and (37), (32)-(34) simplify to:

$$1=E_{\hat{e}}\frac{u'\left(\hat{c}_1\left(x\right)\right)}{u'\left(\hat{c}_0\right)}\frac{u'\left(c_0^*\right)}{u'\left(c_1^*\left(x\right)\right)}.$$

Then, $\hat{K}_1\left(1-\hat{s}\right)+\int_{i\in[0,1]}\hat{S}_1\left(i\right)di\gtrless K_1^*\left(1-s^*\right)+\int_{i\in[0,1]}S_1^*\left(i\right)di$, with $\hat{s}\in(0,1)$, implies $\frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)}\lessgtr\frac{u'\left(c_0^*\right)}{u'\left(c_1^*(x)\right)}$, irrespective of the value of $\hat{e}$. Contradiction. Hence, the only interior solution to (32)-(34) is $\left\{K_1^*,s^*,\{s^*K_1^*\}_i\right\}$ for $s^*\in(0,1)$. In addition, at T^* the local second order sufficient conditions are satisfied. To see this, consider the sub-optimization problem associated with the choice of $\{S_1\left(i\right)\}_i$ and K_1 for given e. In the symmetric equilibria we are considering, expected returns are the same for all stocks and we can restrict attention to the choice of S_1, where $S_1\left(i\right)=S_1$ for all $i=[0,1]$. The elements of the Hessian, H_U, for this problem are:

$$U_{BB}\left(\hat{e}\right)=u''\left(c_0^*\right)+E_{\hat{e}}u''\left(c_1^*\left(x\right)\right)\left(1+D^*-\tau_S^*\left(x\right)\right)^2\leq 0,$$

$$U_{KK}\left(\hat{e}\right)=u''\left(c_0^*\right)+E_{\hat{e}}u''\left(c_1^*\left(x\right)\right)\left(1+x\right)^2\left(1-\tau_P^*\left(x\right)\right)^2\leq 0,$$

$$U_{BK}\left(\hat{e}\right)=u''\left(c_0^*\right)+E_{\hat{e}}u''\left(c_1^*\left(x\right)\right)\left(1+D^*-\tau_S^*\left(x\right)\right)\left(1+x\right)\left(1-\tau_P^*\left(x\right)\right),$$

where U_{xy} denotes a cross-partial derivative with respect to the variables x, y.Under (35)-(36), $|H_U|=0$. Hence, the Hessian is negative semi-definite irrespective of the value of $\hat{e}$. We now consider values of $\hat{K}_1,\hat{S}_1$ that are not interior. The Inada conditions exclude non-interior solutions that result from the non-negativity constraint on time 0 consumption being binding. Hence, there are two candidate non-interior solutions: $\hat{K}_1=0$ and $\hat{S}_1>0$, and $\hat{K}_1>0$ and $\hat{S}_1=\bar{B}$. In both cases, of the two Euler equations for K_1 and S_1, one holds with equality and the other as a strict inequality. Under T^*:

$$1=E_{\hat{e}}\frac{u'\left(\hat{c}_1\left(x\right)\right)}{u'\left(\hat{c}_0\right)}\frac{u'\left(c_0^*\right)}{u'\left(c_1^*\left(x\right)\right)},\tag{38}$$

$$1>E_{\hat{e}}\frac{u'\left(\hat{c}_1\left(x\right)\right)}{u'\left(\hat{c}_0\right)}\frac{u'\left(c_0^*\right)}{u'\left(c_1^*\left(x\right)\right)}.\tag{39}$$

Moreover, (34) implies $\hat{s}=0$. This is a contradiction, since (38) and (39) clearly cannot hold at the same time. Then, K_1^*,S_1^* are globally optimal irrespective of the value of $\hat{e}$ for some $s^*\in[0,1)$. Moreover, at K_1^*,S_1^*, $\rho^*\left(x\right)$ implies $\hat{e}=1$ since the constrained-efficient allocation is incentive compatible. Hence, $\left\{1,K_1^*,s^*,\{s^*K_1^*\}_i\right\}$ is optimal for the agent given the initial endowment K_0, the tax system T^*, and the distribution policy $d\left(x\right)^*$, which implies expected return process D^*. It follows that the resulting allocation, $\left\{c_0^*,1,K_1^*,s^*,\{s^*K_1^*\}_i,c_1^*\left(x\right)\right\}$, jointly with the distribution policy $d^*\left(x\right)$ and the resulting expected return process D^* constitute a competitive equilibrium, according to definition 8. ■

The proof proceeds as the one for proposition 4. The setting of marginal taxes ensures that the entrepreneurs' Euler equations (32)-(34) are satisfied as an equality at any $s^* \in [0,1)$ for distribution policy $d^*(x)$, and that local second order sufficient conditions are also satisfied. It follows that the only interior solution to the entrepreneurs' optimization problem is $\{1, K_1^*, S_1^*(i)\}$ for any $s^* \in [0,1)$. In addition, it ensures that the allocation is globally optimal because it rules out any corner solutions to the entrepreneurs' investment and portfolio problems, irrespective of the level of effort. Lastly, the setting of $\rho^*(x)$ ensures high effort is optimal at the appropriate level of capital and portfolio choices.[10]

The properties of the optimal tax system can be derived from (35)-(37). First:

$$E_1 \tau_P^*(x) = 1 - E_1 \left[\frac{u'(c_0^*)}{\beta (1+x) u'(c_1^*(x))} \right], \tag{40}$$

so that $E_1 \tau_P^*(x) > 0$ if $IW_K > 0$ and $E_1 \tau_P^*(x) < 0$ if $IW_K < 0$. In addition, $\tau_P^*(\bar{x}) - \tau_P^*(\underline{x}) < 0$ when $IW_K > 0$ and $\tau_P^*(\bar{x}) - \tau_P^*(\underline{x}) > 0$ when $\mathrm{IW}_K < 0$ from:

$$\frac{u'(c_0^*)}{\beta (1+\underline{x}) u'(c_1^*(\underline{x}))} - \frac{u'(c_0^*)}{\beta (1+\bar{x}) u'(c_1^*(\bar{x}))} = \tau_P^*(\bar{x}) - \tau_P^*(\underline{x}),$$

since $\mathrm{IW}_K \tilde{} \ (1+\underline{x}) u'(c_1^*(\underline{x})) - (1+\bar{x}) u'(c_1^*(\bar{x}))$. Lastly, by (36):

$$1 + E_1 x - E_1 \tau_P^*(x) - E_1 x \tau_P^*(x) - E_1 \tau_S^*(x) = E_1 \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{41}$$

This implies $E_1 \tau_S^*(x) = -E_1 \tau_P^*(x) - E_1 x \tau_P^*(x) = -E_1 \tau_P^*(x) E_1(1+x) - Cov_1(x, \tau_P^*(x))$. If $IW_K \gtrless 0$, $Cov_1(x, \tau_P^*(x)) \lesseqgtr 0$ and $E_1 \tau_P^*(x) \gtrless 0$, as discussed above. Hence, the sign of $E_1 \tau_S^*(x)$ is typically ambiguous.

The optimal tax system does not pin down the equilibrium value of s^*. By (32), for $s^* \in [0,1)$, the tax system ensures that entrepreneurs find it optimal to choose K_1^*.

For the benchmark parameterization, the pattern of optimal taxes on earnings and stocks as a function of the coefficient of relative risk aversion, is displayed in figure 5:

The tax system described in proposition 9 embeds a prescription for *double taxation of income from entrepreneurial capital*: at the firm level thought τ_P^*, and at the level of external investors, through τ_S^*. This property is jointly determined by the distribution policy and the tax system, since external investors receive a share of earnings *after* tax.

[10] The constrained-efficient allocation can equivalently be implemented with a marginal tax on tax on capital $\tau_K(x)$ that satisfies (22) and with distribution policy: $1 + d(x) = 1 + x - \tau_K(x)$ and dividend process $1 + D(i, \tilde{x}) = 1 + \tilde{x} - \tau_K^*(\tilde{x})$, so that $1 + D(i) = 1 + E_1(x)$, since $E_1 \tau_K^*(x) = 0$. The optimality of double taxation of entrepreneurial earnings can be derived with a similar reasoning.

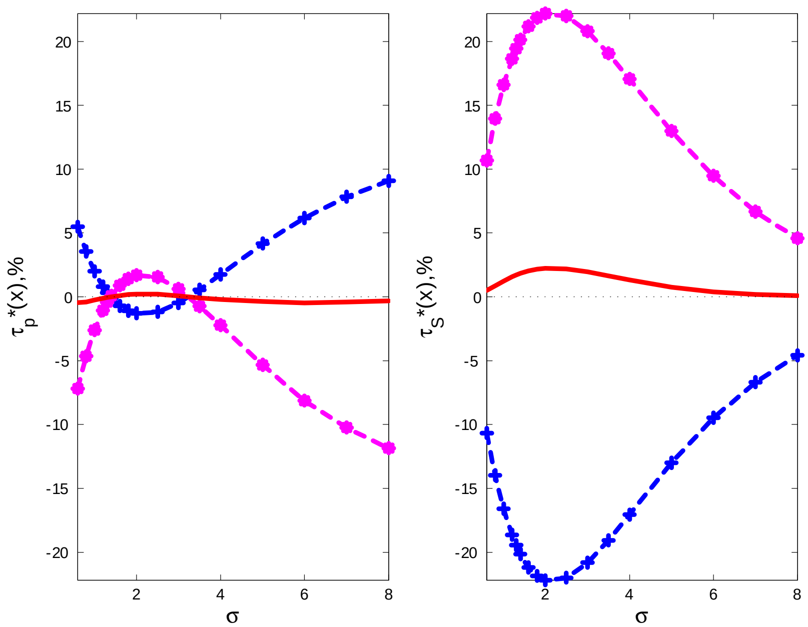


Figure 5: Optimal marginal taxes on earnings and stocks. Benchmark parameterization.

We now explore whether this feature of the tax system necessary to implement the constrained-efficient allocation.

First, observe that taxation of stock earnings received by external investors is *required* to ensure that entrepreneurs choose S_1^*. Given that returns on external stocks are uncorrelated with idiosyncratic capital returns for each entrepreneur, the wedge on equity holdings is positive and equal to the aggregate intertemporal wedge IW, since by (40) and (41):

$$\beta E_1 \left(1 + D\left(i, \tilde{x}\right)\right) E_1 u'\left(c_1^*\left(x\right)\right) - u'\left(c_0^*\right) = \text{IW} > 0.$$

Following the reasoning in section 2.1, absent a marginal tax on stock holdings, entrepreneurs would have an incentive to increase holdings of stocks and reduce effort.

To explore whether it is necessary to tax distributed earnings at the firm level, we allow the marginal tax on distributed earnings, $\tau_d\left(x\right)$ to differ from the marginal tax on retained earnings, $\tau_P\left(x\right)$. Then, the distribution policy is: $1+d\left(x\right) = \left(1+x\right)\left(1-\tau_d\left(x\right)\right)$ and the candidate tax system can be written as $T\left(K_1, s, \left\{S_1\left(i\right)\right\}_i\right) = \tau_P\left(x\right)\left(1-s\right)K_1 + \tau_d\left(x\right)sK_1 + \tau_s\left(x\right)\int_i S_1\left(i\right)di + \rho\left(x\right)$. Setting $\tau_d\left(x\right) = 0$ for $x \in X$ avoids double taxation of entrepreneurial earnings. By (32) and (34), $1 + d\left(x\right) = \left(1+x\right)$ implies: $u'\left(c_0^*\right) - \beta E_1\left(1+x\right)u'\left(c_1^*\left(x\right)\right) < 0$ and $s = 0$, if the individual intertemporal wedge is positive. It implies: $u'\left(c_0^*\right) - \beta E_1\left(1+x\right)u'\left(c_1^*\left(x\right)\right) > 0$ and $s = 1$, if the intertemporal wedge is negative. However, an entrepreneur's optimal choice of capital is undetermined at $s = 1$, since her utility does not depend on K_1 in this case. Moreover, at $s = 1$, an entrepreneur does not have any gains from effort, so $e = 0$. Hence, a tax system that implements the constrained-efficient allocation must ensure that the equilibrium value of s is strictly smaller than 1, and this is not possible if distributed earnings are not taxed at the firm level when the individual intertemporal wedge is negative.

More in general, the class of tax systems $T\left(K_1, s, \left\{S_1\left(i\right)\right\}_i\right)$ that rules out $s = 1$ as a possible solution to the entrepreneurs' problem can be characterized as follows.

Proposition 10. *In any competitive equilibrium under a tax system,* $T\left(K_1, s, \left\{S_1\left(i\right)\right\}_i\right) = \tau_P\left(x\right)\left(1-s\right)K_1 + \tau_d\left(x\right)sK_1 + \tau_s\left(x\right)\int_i S_1\left(i\right)di + \rho\left(x\right)$, *and distribution policy* $1 + d\left(x\right) = \left(1+x\right)\left(1-\tau_d\left(x\right)\right)$, $s \in [0, 1)$ *if and only if:*

$$E_{\hat{e}}\left(1 + d\left(x\right)\right)u'\left(c_1\left(x\right)\right) \geq E_{\hat{e}}\left(1+x\right)\left(1-\tau_P\left(x\right)\right)u'\left(c_1\left(x\right)\right). \tag{42}$$

Proof. Suppose that the distribution policy is $\hat{d}\left(x\right)$ and that $E_{\hat{e}}\left(1+\hat{d}\left(x\right)\right)u'\left(c_1\left(x\right)\right) \neq E_{\hat{e}}\left(1+x\right)\left(1-\tau_P\left(x\right)\right)u'\left(c_1\left(x\right)\right)$ for some tax system where (32) holds with equality at $\hat{\tau}_P\left(x\right)$. Denote the corresponding competitive equilibrium allocation with $\left\{\hat{K}_1, \hat{s}, \left\{\hat{S}_1\left(i\right)\right\}_i, \hat{e}, \hat{c}_1\left(x\right)\right\}$, with $\hat{K}_1 > 0$. If $E_{\hat{e}}\left(1+\hat{d}\left(x\right)\right)u'\left(c_1\left(x\right)\right) > E_{\hat{e}}\left(1+x\right)\left(1-\tau_P\left(x\right)\right)u'\left(c_1\left(x\right)\right)$, for some

$0 < \hat{s} < 1$, we can write:

$$\begin{aligned} 0 &= -u'(\hat{c}_0)(1-\hat{s}) + \beta E_{\hat{e}}\left[(1+x)(1-\hat{\tau}_P(x)) - \left(1+\hat{d}(x)\right)\hat{s}\right]u'(\hat{c}_1(x)) \\ &< -(1-\hat{s})\left[u'(\hat{c}_0) - \beta E_{\hat{e}}\left(1+\hat{d}(x)\right)u'(\hat{c}_1(x))\right], \end{aligned}$$

which implies $0 > u'(\hat{c}_0) - \beta E_1 \left(1+\hat{d}(x)\right) u'(\hat{c}_1(x))$. But by (34), $\hat{s} = 0$. Contradiction. Similarly, if $E_{\hat{e}}\left(1+\hat{d}(x)\right)u'(c_1(x)) < E_{\hat{e}}(1+x)(1-\tau_P(x))u'(c_1(x))$ for some $0 < \hat{s} < 1$:

$$\begin{aligned} 0 &= -u'(\hat{c}_0)(1-\hat{s}) + \beta E_{\hat{e}}\left[(1+x)(1-\hat{\tau}_P(x)) - \left(1+\hat{d}(x)\right)\hat{s}\right]u'(\hat{c}_1(x)) \\ &> -(1-\hat{s})\left[u'(\hat{c}_0) - \beta E_{\hat{e}}\left(1+\hat{d}(x)\right)u'(\hat{c}_1(x))\right]. \end{aligned}$$

Then, $u'(\hat{c}_0) - \beta E_1 \left(1+\hat{d}(x)\right) u'(\hat{c}_1(x)) > 0$, which by (34) implies $\hat{s} = 1$. Contradiction. ■

By proposition 10, the expected discounted value of distributed earnings must be greater than the expected discounted value of retained earnings after tax to ensure that $s < 1$ in a competitive equilibrium under a tax system $T\left(K_1, s, \{S_1(i)\}_i\right)$. For this condition to be verified at the constrained-efficient allocation, it must be that $\tau_P(x) \geq \tau_d(x)$ if $u'(c_1^*(x))(1+x) > u'(c_1^*(x'))(1+x')$ for $x, x' \in \{\underline{x}, \bar{x}\}$. Then, since $u'(c_1^*(\underline{x}))(1+\underline{x}) \gtrless u'(c_1^*(\bar{x}))(1+\bar{x})$ for $\mathrm{IW}_K \gtrless 0$, this implies $\tau_P(\bar{x}) \leq \tau_d(\bar{x})$ and $\tau_P^*(\underline{x}) \geq \tau_d(\underline{x})$ for $\mathrm{IW}_K > 0$ and $\tau_P(\bar{x}) \geq \tau_d(\bar{x})$ and $\tau_P(\underline{x}) \leq \tau_d(\underline{x})$ for $\mathrm{IW}_K < 0$.

The rationale for this result is simple. When $\mathrm{IW}_K > 0$, entrepreneurs have an incentive to increase holdings of their own capital and reduce effort at the constrained-efficient allocation. A way to discourage this is to make external capital a good hedge. This is achieved by making dividend payouts greater in the good state and smaller in the bad state. Conversely, when $\mathrm{IW}_K < 0$, entrepreneurs have an incentive to reduce holdings of their own capital and effort. To avoid an outcome in which entrepreneurs retain too little ownership, the tax system must make external capital a bad hedge, by making dividend payments higher in the bad state and lower in the good state.

The first order necessary conditions for K_1 can be rewritten as:

$$\begin{aligned} 0 = &-(1-s)\left\{u'(c_0^*) - \beta E_1\left[(1+x)(1-\tau_P^*(x))u'(c_1^*(x))\right]\right\} \\ &+\beta s E_1\left[(1+x)(\tau_d(x) - \tau_P^*(x))u'(c_1(x))\right]. \end{aligned}$$

Then, if $\tau_P^*(x)$ satisfies (35), it must be that $E_1\left[(1+x)(\tau_d(x) - \tau_P^*(x))u'(c_1(x))\right] = 0$, to ensure that K_1^* is chosen if $s > 0$. This also satisfies (42), and ensures that $s < 1$.

This argument implies that it is indeed *necessary* for distributed earnings, as well as retained earnings, to be taxed at the firm level to implement the constrained-efficient allocation. Hence, entrepreneurial capital is subject to *double taxation* in the optimal tax system.

We have so far assumed that entrepreneurs' holdings of financial securities and equity are observed by the government. However, given that the optimal tax system is linear in asset levels, individual tax payments are independent from the level of asset holdings. This implies that the government does not need to observe entrepreneurs' portfolios to administer the optimal tax system, if financial securities are traded via competitive intermediaries. The intermediaries, in an arrangement similar to the one in place for consumption taxes in the US, can collect asset taxes *at the source* according to the specified schedule, since marginal taxes are conditional on publicly observable information. We explore this market structure in the following section.

3.3. Private Insurance Contracts

We now construct an implementation with private insurance contracts. We assume that there are a continuum of identical insurance companies that behave competitively. Insurance companies are risk neutral agents that don't exert any effort. Each insurance company writes contracts with a continuum of entrepreneurs. The insurance companies are owned by the entrepreneurs and their profits are transferred to the entrepreneurs in each period.

Events occur according to the following timing. At time 0, insurance companies offer incentive compatible insurance contracts to the entrepreneurs, denoted with $\mathcal{C} = \{P, R(\underline{x}), R(\bar{x})\}$, where P is the premium paid at time 0 and $R(x)$ is the state contingent transfer at time 1. Entrepreneurs can only purchase one insurance contract. In addition, entrepreneurs buy bonds B_1 which pay a risk-free interest r, and they invest in capital K_1. They then exert effort. In period 1, x is realized, entrepreneurs receive insurance payments and the government levies taxes. Insurance companies are liquidated and their liquidation value is rebated to the entrepreneurs. The entrepreneurs then consume.

The informational structure is as follows. The level of investment K_1 and x are public information. We assume that insurance companies and the government do not observe B_1. We restrict attention to candidate tax systems of the form: $T(K_1, B_1, x) = \rho(x) + \tau_B(x) B_1 + \tau_K(x) K_1$.

The optimal insurance contracts solve the following problem:

$$\Pi = \max_{\left[e, K_1, B_1, P, R(x), B_1^I\right] \in \Phi(K_0)} \left\{ P - B_1^I + \frac{B_1^I (1+r) - \left[\pi(e) R(\bar{x}) + (1-\pi(e)) R(\underline{x})\right]}{1+r} \right\}, \tag{Problem 5}$$

subject to

$$U\left(e,K_1,B_1;\mathcal{C},T\right)-v(e)\geq U\left(\tilde{e},\tilde{K}_1,\tilde{B}_1;\mathcal{C},T\right)-v(\tilde{e}),\ \text{for}\ \left[\tilde{e},\tilde{P},\tilde{R}\left(x\right),\tilde{K}_1,\tilde{B}_1\right]\in\Phi\left(K_0\right), \tag{43}$$

$$P-\frac{\pi\left(e\right)R\left(\bar{x}\right)+\left(1-\pi\left(e\right)\right)R\left(\underline{x}\right)}{1+r}=0, \tag{44}$$

where

$$\Phi\left(K_0\right)\equiv\left\{\begin{array}{c}\left[e,K_1,B_1,P,R\left(x\right)\right]:K_1\in\left[0,K_0+\bar{\Pi}_0-P-B_1\right],B_1\geq\bar{B},\\ R\left(x\right)\geq K_1\left(1+x-\tau_K\left(x\right)\right)+B_1\left(1+r-\tau_B\left(x\right)\right)-\rho\left(x\right)+\bar{\Pi}_1.\end{array}\right\} \tag{45}$$

$$\begin{aligned}U\left(e,K_1,B_1;\mathcal{C},T\right) &= u\left(K_0+\bar{\Pi}_0-P-K_1-B_1\right)\\ &\quad+\beta E_e u\left(K_1\left(1+x-\tau_K\left(x\right)\right)+B_1\left(1+r-\tau_B\left(x\right)\right)+R\left(x\right)-\rho\left(x\right)+\bar{\Pi}_1\right)\end{aligned}$$

$\bar{B}$ is the natural debt limit, and $\bar{\Pi}_t$ denotes aggregate profits from the insurance sector in period $t=0,1$. Each individual insurance company takes $\bar{\Pi}_t$ as given.

Constraint (43) is the incentive compatibility constraint. It requires that the effort, capital and bond allocation specified by the contract is preferred by the agent to any other feasible effort, capital and bond allocation. Insurance companies cannot observe effort and bond holdings, but can induce a particular allocation which is incentive compatible. The entrepreneur takes the tax system, the terms of the insurance contract and the bank's liquidation value as given. Constraint (44) is the zero profit condition imposed on insurance companies. The set $\Phi\left(K_0\right)$ describes feasible allocations and contracts. The feasibility requirements reflect the non-negativity constraints in the agents problem.

We assume that entrepreneurs and insurance companies buy bonds from financial intermediaries that collect taxes on bonds at the source. The cash flow of financial intermediaries is denoted with F_t for $t=0,1$, with $F_0=B_1+B_1^I$, $F_1=-B_1\left(1+r\right)$ $-B_1^I\left(1+r-\left[\pi\left(e\right)\tau_B\left(\bar{x}\right)+\left(1-\pi\left(e\right)\right)\tau_B\left(\underline{x}\right)\right]\right)-\left[\pi\left(e\right)\tau_B\left(\bar{x}\right)+\left(1-\pi\left(e\right)\right)\tau_B\left(\underline{x}\right)\right]B_1$.

Definition 11. *A competitive equilibrium with insurance contracts is given by an initial endowment of capital K_0 for the entrepreneurs, an allocation $\{e,K_1,B_1\}$, insurance contracts $\mathcal{C}$, and a tax system $T\left(K_1,B_1,x\right)$ such that:*
i) the allocation and loan contracts $\mathcal{C}$ solve Problem 4 given the tax system;
ii) the bond market clears, $B_1+B_1^I=0$;
iii) the resource constraint is satisfied in each period.

The first requirement guarantees that the allocation and the corresponding consumption path are optimal for the entrepreneurs, given the tax system and the insurance contracts, since the allocation and contracts are incentive compatible. Insurance companies are optimizing and make zero profits in equilibrium given that they solve Problem 4. In a competitive equilibrium, financial intermediaries obtain a zero cash-flow in each period.

We define the optimal tax system as the one that implements the allocation that solves Problem 1, denoted with $e^* = 1$, K_1^*, c_0^*, $c_1^*(\underline{x})$, $c_1^*(\bar{x})$, in a competitive equilibrium.

Proposition 12. *Let $\tau_K^*(x)$ and $\tau_B^*(x)$ satisfy (22) and (23) and set $\rho^*(x) = 0$ for $x = \underline{x}, \bar{x}$. Then, the tax system $T^*(K_1, B_1, x) = \rho^*(x) + \tau_K^*(x) K_1 + \tau_B^*(x) B_1$ implements the allocation $e^* = 1$, K_1^*, c_0^*, $c_1^*(\underline{x})$, $c_1^*(\bar{x})$ in the economy with insurance contracts and unobservable bond holdings with $B_1^* = B_1^{I*} = 0$ and $r = E_1(x)$.*

Proof. We construct a competitive equilibrium in which the allocation is $e^* = 1$, K_1^*, c_0^*, $c_1^*(\underline{x})$, $c_1^*(\bar{x})$, bond holdings are $B_1^* = 0$ and the equilibrium rate of return on bonds is $r = E_1(x)$. In this equilibrium, $P^* = B_1^I = 0$. To characterize the optimal insurance contracts, we consider a relaxed version of Problem 5, in which the incentive compatibility constraint (43) is replaced by the set of constraints:

$$\begin{aligned} & (\pi(1) - \pi(0)) \left[u\left(K_1(1 + \bar{x} - \tau_K(\bar{x})) + R(\bar{x}) + B_1(1 + r - \tau_B(\bar{x})) - \rho(\bar{x}) + \bar{\Pi}_1\right)\right. \\ & \left. -u\left(K_1(1 + \underline{x} - \tau_K(\underline{x})) + R(\underline{x}) + B_1(1 + r - \tau_B(\underline{x})) - \rho(\underline{x}) + \bar{\Pi}_1\right)\right] \\ \geq\ & \Delta v, \end{aligned} \tag{46}$$

$$\begin{aligned} & u'\left(K_0 + \bar{\Pi}_0 - P - K_1 - B_1\right) \\ =\ & E_1 u'\left(K_1(1 + x - \tau_K(x)) + R(x) + B_1^i(1 + r - \tau_B(x)) - \rho(x) + \bar{\Pi}_1\right)(1 + x - \tau_K(x)), \end{aligned} \tag{47}$$

$$\begin{aligned} & u'\left(K_0 + \bar{\Pi}_0 - P - K_1 - B_1\right) \\ =\ & E_1 u'\left(K_1(1 + x - \tau_K(x)) + R(x) + B_1(1 + r - \tau_B(x)) - \rho(x) + \bar{\Pi}_1\right)(1 + r - \tau_B(x)). \end{aligned} \tag{48}$$

These constraints are the first order conditions for an agent's optimization problem embedded in constraint (43). We refer to the contracting problem under (46)-(48) as Problem 6. We construct a solution to Problem 6 under a candidate optimal tax system and then we show that this solution also solves Problem 5. Then, (47) and (48) will be satisfied at c_0^*, $c_1^*(x)$ and K_1^* and c_0^* are feasible for $B_1^* = 0$. Let $P^* = B_1^I = 0$ and set $R^*(x)$ satisfy:

$$c_1^*(x) = K_1^*(1 + x - \tau_K^*(x)) + R^*(x) + B_1^*(1 + r - \tau_B^*(x)) - \rho^*(x). \tag{49}$$

$R^*(x)$ is clearly feasible for the insurance companies, since (46)-(**??**) are satisfied at the constrained-efficient allocation. In addition, $\bar{\Pi}_1 = 0$. We need to show that it is indeed optimal. The insurers' problem at $\tau_K^*(x)$ and $\tau_B^*(x)$ and $\rho^*(x)$ can be rewritten with a change of variables as:

$$\max_{e\in\{0,1\},K_1\in[0,K_0],c_0\geq 0,c_1(x)\geq 0,B_1}\left\{\frac{E_e\left[K_1\left(1+x-\tau_K^*(x)\right)-c_1(x)\right]}{1+r}-\left(c_0+K_1-K_0\right)\right\},$$

by substituting the agents budget constraints in each period, since $\bar{\Pi}_t$ is taken as given. The level of B_1 does not matter for the value of this objective. Let $I_0 = K_0 - K_1 - c_0$ and $I_1(x) = K_1(1+x) - c_1(x)$. Consider the problem:

$$\Gamma(K_0) = \max_{e\in\{0,1\},K_1\in[0,K_0],c_0\geq 0,c_1(x)\geq 0}\left\{I_0+\frac{E_e\left[I_1(x)-\tau_K^*(x)\right]}{1+r}\right\} \qquad \text{(Problem 7)}$$

subject to

$$I_t \geq 0,\ t=0,1,$$
$$u(c_0) - v(e) + \beta E_e u(c_1(x)) \geq \bar{U}, \qquad (50)$$

and (3). The variables I_t, for $t = 0, 1$ are economy resources net of consumption in each period. Hence, for $1/(1+r) = 1/E_1(1+x)$ Problem 7 can be interpreted as a dual planning problem in which the planner minimizes the resource cost of providing a consumption allocation to the agent, subject to an incentive compatibility constraint and a participation constraint.

We now proceed in several steps. First, we show that for $\bar{U} = U^*(K_0)$, the solution of this problem is $e^* = 1, K_1^*, c_0^*, c_1^*(x)$. Suppose not, let $\left[\tilde{e},\tilde{K}_1,\tilde{c}_0,\tilde{c}_1(x)\right]$ solve Problem 7 at $1/(1+r) = 1/E_1(1+x)$ and $\bar{U} = U^*(K_0)$ with $\left[\tilde{e},\tilde{K}_1,\tilde{c}_0,\tilde{c}_1(x)\right] \neq [1, K_1^*, c_0^*, c_1^*(x)]$. $\left[\tilde{e},\tilde{K}_1,\tilde{c}_0,\tilde{c}_1(x)\right]$ is clearly feasible for Problem 1. Moreover, by (50) it attains the maximum for Problem 1. Given that Problem 1 has a strictly concave objective with a convex constraint set, the solution is unique. Hence, $\left[\tilde{e},\tilde{K}_1,\tilde{c}_0,\tilde{c}_1(x)\right]$ must solve Problem 1. Contradiction. Then, $[1, K_1^*, c_0^*, c_1^*(x)]$ solves Problem 7, which is a relaxed version of Problem 6, since (3) is the incentive compatibility constraint when B_1 is observable. In addition, since $1, K_1^*, c_0^*, c_1^*(x)$ and $B_1^* = 0 = \bar{\Pi}_0 = P^*$ satisfy (47) and (48) under the tax system $T^*(K_1, B_1, x) = \tau_K^*(x)K_1 + \tau_B^*(x)B_1$, K_1^*, B_1^* and $e^* = 1$, given P^* and $R^*(x)$, they solve Problem 6, since they are optimal for Problem 7, which is less constrained. To see that they also solve Problem 5, note that following the arguments the the proof of proposition 4, we can show that an agent's local second order conditions are satisfied and that $1, K_1^*, c_0^*, c_1^*(x)$ and $B_1^* = 0$ are globally optimal. In addition, $R^*(x)$ satisfies (44). Hence, $1, K_1^*, c_0^*, c_1^*(x)$ and $B_1^* = 0$ are feasible for problem 5, given P and $R^*(x)$, and will be optimal for Problem 5, since they are the solution to a relaxed problem. ■

This property has important implications for the role of taxes in implementing allocations. Under this market structure, entrepreneurs entertain an exclusive relationship with a insurance company. If the entrepreneurs bond holding were observed by the insurer, the optimal contract would implement the constrained efficient allocation in a competitive equilibrium where insurance companies make zero profits. Since bond holdings are not observed by insurers, it is necessary for the government to set marginal asset taxes to influence the entrepreneurs' intertemporal choice. Even if the government has the same informational constraints as private insurance companies, namely it cannot observe entrepreneurial financial asset holdings, it can influence the portfolio choices of entrepreneurs through the tax system. Proposition 12 shows that, by appropriately setting marginal asset taxes, the government can relax the more severe incentive compatibility constraint that arises in the contracting problem between private insurance companies and entrepreneurs, due entrepreneurs' unobserved holdings of financial assets, thus enabling private insurance contracts to implement the constrained-efficient allocation with observable consumption.

Bizer and DiMarzo (1999) derive a related result in a standard moral hazard model in which agents may borrow. They show that as long as debt repayments can be made state contingent, it is possible to implement the constrained-efficient allocation with observable savings, even if borrowing is unobserved by the principal, who designs the incentive-compatible transfer (salary) policy. In their setting, it is important that agents borrow, rather than save. Only in this case can the return be made state contingent. This requirement does not hold in our implementation, since the government can set marginal bond taxes to be state contingent.

These findings are related to Golosov and Tsyvinski (2004), who analyze fiscal implementations in a Mirrleesian economy with unobserved consumption. They show that private insurance contracts do not implement constrained-efficient allocations in that setting, because private insurers fail to internalize the effect of the contracts they offer on the equilibrium price of unobservable bond trades. A linear tax on capital can instead ameliorate this externality. However, their tax system implements the constrained-efficient allocation with unobservable consumption. In the market structure presented here, the tax system implements the constrained-efficient allocation with observable consumption, despite the fact that in the market economy the bond holdings are unobserved by the government and by private insurance firms.

4. Concluding Remarks

This paper analyzes optimal taxation of entrepreneurial capital. The contribution of this analysis is twofold. First, we characterize the properties of constrained-efficient allocations in private information economies with idiosyncratic capital returns.[11] We show that the intertemporal wedge on entrepreneurial capital can be positive or negative. It is negative when the spread in capital returns is sufficiently large or the spread in consumption across states at the constrained-efficient allocation is sufficiently small. A negative intertemporal wedge signals that more capital has a positive effect on incentives. This can occur since the returns from effort are increasing in capital. Second, we derive the properties of optimal taxes on entrepreneurial capital as well as on other financial assets. We show that marginal asset taxes depend on the *correlation* of their returns with idiosyncratic uncertainty. We also consider whether entrepreneurial capital earnings distributed to outside investors should be taxed at the firm level. We find that entrepreneurial capital should be taxed at the firm level and again when it accrues to outside investors in the form of stock returns. This generates a theory of optimal differential asset taxation and provides a foundation for the double taxation of capital earnings.

The empirical public finance literature has documented substantial differences in the tax treatment of different forms of capital income. Specifically, interest income is taxed at a higher rate than stock returns, as discussed in Gordon (2003), while dividends are taxed at a higher rate than realized capital gains. As documented by Gordon and Slemrod (1988), the higher marginal tax rate on interest income is a stable property of empirical tax systems in many industrialized economies. Poterba (2002) has documented a strong response of household portfolio composition to this differential tax treatment. Personal and corporate tax rates on capital income are also different. Auerbach (2002) finds that firms's investment decisions appear to be sensitive to the taxation of dividend income at the personal level and their choice of organization form is responsive to the differential between corporate and personal tax rates.

In the economy studied in this paper, the optimal tax system implements the constrained-efficient allocation by influencing portfolio choice and sales of private equity by entrepreneurs. Differential tax treatment of different asset classes is essential to achieve this goal. We also show that capital income taxes are essential to implement constrained-efficient allocations when entrepreneurs can buy private insurance contracts but insurers cannot observed entrepreneurs' asset holdings. This points to an important complementarity between private contracting and taxation.

The incentive problem that arises with entrepreneurial capital arguably also applies to top executives who hold company stock and other assets. Hence, this analysis could be adapted to such a setting. A quantitative version of the model can be used to provide an assessment of empirical tax systems. We leave these extensions for future work.

[11] This class of environments has not been studied in the recursive contracting literature. An exception is Kahn and Ravikumar (1999). They focus on an implementation with financial intermediaries and rely on numerical simulations. They do no provide an analytical characterization of the wedges associated with the constrained-efficient allocation.

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